**Unit 10: Sequences**

**Lesson 2: Geometric sequence**

**Objectives:**

* **I can identify a geometric sequence**
* **I list terms in a geometric sequence**
* **I can identify the common ratio in a geometric sequence**
* **I can calculate the nth term of a geometric sequence**

**Vocabulary:** Sequence, geometric sequence, common ratio, initial term, nth term, recursive form, explicit form.

**Agenda:**

* video
* practice
* Application

**Focus Questions:**

1. **What is a geometric sequence?**
2. **How do we write an explicit rule and a recursive rule for a geometric sequence?**
3. **What is the connection between geometric sequences and exponential functions?**

**Homework: Worksheet 10-2**

**Online support**

* <https://www.khanacademy.org/math/algebra/sequences/introduction-to-geometric-sequences/v/geometric-sequences-introduction>
* <https://youtu.be/C7tE26CDI2M>
* <https://www.youtube.com/watch?v=IGFQXInm-co>
* <https://www.youtube.com/watch?v=pwUdYEwT9kY>

**Warm UP**

Two important types of sequences are **arithmetic** sequences I Lesson 1) and **geometric** sequences ( Lesson 2). Check whether the sequence is arithmetic or geometric, and fill in the missing number in each sequence.

|  |  |  |
| --- | --- | --- |
| **sequence** | **Arithmetic( nth rule)** | **Geometric ( nth rule)** |
| 1. 4, 7, 10, \_\_\_\_\_\_\_\_, 16, . . .
 |  |  |
| 1. 2, 4, 8, 16, \_\_\_\_\_\_\_\_, . . .
 |  |  |
| 1. 1, \_\_\_\_\_\_\_\_, 9, 27, 81, . . .
 |  |  |
| 1. 3.5, 6, 8.5, 11, \_\_\_\_\_\_\_\_\_, . . .
 |  |  |
| 1. 8, 12, 18, \_\_\_\_\_\_\_\_\_, 40.5, . . .
 |  |  |
| 1. \_\_\_\_\_\_\_\_, -5.5, -9.5, -13.5, . . .
 |  |  |
| 1. 256, 64, 16, 4, \_\_\_\_\_\_\_\_\_, . . .
 |  |  |
| 1. -4, 8, -16, 32, -64, \_\_\_\_\_\_\_\_\_,
 |  |  |

A **sequence** is a list of numbers in a particular order. Each number in a sequence is called a **term**. The first term is symbolized by , the second term is symbolized by , and so on. There are two major types of explicit sequences, arithmetic and geometric.

**Geometric** – a sequence of numbers such that each number after the first is found by multiplying the previous one by a constant called the **common ratio.**

|  |  |  |
| --- | --- | --- |
| **Geometric** | **Equation Type -** Exponential | **Formula for the nth term:**  **where** **is the initial term and** ***r* is the common ratio** |

Example 1: Identify if each one of the following sequences is a geometric sequence or not. If yes identify$ a\_{1} , r$?

$$1. A=\left\{3, 5, 7, 11, 24\right\} 2. B=\{-5,-10, -20, -40\}$$

Example 2: Find the $a\_{1 } $and common ratio r by dividing two consecutive terms for each of the following geometric sequences.

1. {5, 10, 20, …} 2. { 2, -6, 18, -54,…….}

* $a\_{1}= $ $a\_{1}= $
* r = \_\_\_\_\_\_\_ r = \_\_\_\_\_\_\_
* The next 4 terms are: \_\_\_\_\_\_\_ The next 4 terms are: \_\_\_\_\_\_\_

**Write the general rule for each, then find the 8th term for each.**

Example 3: Identify if the sequence is arithmetic, geometric or neither, and find first five terms.

1.  b. $a\_{n}=-2+3(n-1)$

Example 4:Identify $ a\_{1}, r$ and list the first 5 terms for the following geometric nth rule

 1. $ a\_{n}=2(-4)^{n-1}$

Find the 10th term

Example 5: Write the rule that describes each of the geometric sequences explicitly.

1. 3, 12, 48, 192, … 2. 2, 6, 18, 54, 162,..
* $a\_{1}= $ $a\_{1}= $
* r = \_\_\_\_\_\_\_ r = \_\_\_\_\_\_\_
* General Rule from reference sheet: \_\_\_\_\_\_\_ Gen Rule:\_\_\_\_\_\_\_
* Substitute your parameters:\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_

Example 6: Growing Dot Problem: let’s look at the following example and discuss with your partner:



1. Assuming the sequence continues in the same way, how many dots are there at 3 minutes? 4 min?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| M (minute) | 0 | 1 | 2 | 3 | 4 |
| D (dots) |  |  |  |  |  |

1. List the number of the dots after each minute as a sequence from the beginning to min 4.
2. Identify $a\_{1} and r$ for the sequence and write the $n ^{th}$ term rule for the sequence.

 **Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Homework 10-2 Geometric sequences**

Use The explicit formula is From the reference sheet.

1. Is the following sequence arithmetic or geometric, and what is the common difference (*d*) or the common ratio (*r*) of the sequence? Must justify:
2.  b. 2, 10, 50, … c. 5, 8, 11, 14
3. Identify the common ratio, *r*,and the first term for the following sequence?
4.  b.  c. 

Write a general rule for each:

1. What is the common ratio, *r*, first term, and the first 2terms and use the general rule to find the 20 term for the following geometric sequence

 *an* = 2(5)*n*−1 b.  c. *an* = −2(3)n-1

1. Write the explicit formula and the find the 15th term.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **n** | 1 | 2 | 3 | 4 | 5 | . . .  |
| **Term** | 16 | -8 | 4 | -2 | 1 | . . .  |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **n** | 1 | 2 | 3 | 4 | 5 | . . .  |
| **Term** | 0.01 | 0.06 | 0.36 | 2.16 | 12.96 | . . .  |

1. Write the explicit formula and the find the 10th term.

|  |
| --- |
| 1. Use the following geometric sequence for the problem below:-17, 10.2, -6.12, 3.672, ...
 |
| For the formula that represents the general term \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

|  |
| --- |
| 7)Use the following geometric sequence for the problem below:4, 3.2, 2.56, 2.048, ... Find r first then substitute in |
| For the formula that represents the general term, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
|  |

8)\*\*Find the 10th term of a geometric sequence for which = 108 and r = 3.

First, find the value of . Now find.

Recursive rules:

**Example 1:** Find r and the 5th term of the following recursive rule of the geometric sequence. $a\_{n}=3a\_{n-1} , a\_{1}=4$

**Example 2: w**rite an equation for the nth term of the geometric sequence. 2, 6, 18, 54, 162, . . .

 In this sequence, = \_\_\_\_\_\_\_ and r = \_\_\_\_\_\_\_.

Explicitly Recursively

 $ a\_{n}=\left(r\right)a\_{n-1} , a\_{1}=$

**Example 3**: Find the explicit formula for the sequence defined by the recursive formula and identify if the sequence is arithmetic or geometric:

1. *a*1 = -2, *an* = -5(*an*-1) b. *a*1 = 4, *an* = *an*-1 - 3

**Example 4:**



Example 6: A ball is dropped from a height of 27 feet. On each rebound the ball rises to a height of two-thirds that from which it fell.

1. Make a table of values that express y the height of the ball after x number of bounces.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of bounces x | 0 | 1 | 2 | 3 | 4 | 5 |
| Height y |  |  |  |  |  |  |

(**b**) How far will the ball rise on the fourth rebound? [*Show all work.*]

**Exercise 9:** For the geometric sequence defined by

 $a\_{1}=16$ and $a\_{n}=a\_{n-1}(\frac{1}{2})$

List and plot the first 6 terms on the grid below

Graphs of geometric sequences will looks familiar. How so?

**Working backwards: Find a term given another term and the ratio**

**Exercise 10:** Find the explicit formula for a sequence where r = 2 and .

The formula says that we need to know the first term and the common ratio. We have r, but do not know a1. However, we have enough information to find it. We know that when n = 12, the 12th term in the sequence is 14336.

 Find $a\_{7}$